

ANALYSIS QUALIFYING EXAM

JUNE, 2014

), X a measure space and μ a finite measure. Answer all 4 questions. In your proofs, y

$f : X \rightarrow \mathbb{R}$ measurable. Show that

$$\int_X |f(x)|^p d\mu(x) = \int_0^\infty p t^{p-1} \mu(\{x : |f(x)| > t\}) dt,$$

where $\mu(\{x : |f(x)| > t\}) = \mu\{x : |f(x)| > t\}$.

Exercise 2. (30 points.)

- (1) Prove that not every subset of $[0, 1]$ is Lebesgue measurable
- (2) Let $f_n : [0, 1] \rightarrow \mathbb{R}$ be a sequence of Lebesgue measurable functions. Prove that the set $E = \{x : \lim_n f_n(x) \text{ exists}\}$ is Lebesgue measurable

Exercise 3. (30 points.)

Let X, Y be Banach spaces. If $T : X \rightarrow Y$ is a linear map such that $\|Tf\|_Y \leq C \|f\|_X$ for all $f \in X$ then T is bounded.

Exercise 4. (30 points.)

Let (X, \mathcal{M}, μ) be a finite measure space. For each of the following claims prove or give a counter example:

- (1) If a sequence (f_n) of real valued measurable functions on X converges μ a.e., then (f_n) converges in measure.
- (2) If a sequence (f_n) of real valued measurable functions on X converges in measure, then (f_n) converges μ a.e.
- (3) If a sequence (f_n) of real valued measurable functions on X is Cauchy in $L^1(\mu)$, then (f_n) converges in measure.