Algebra Qualifying Examination, June 2019

Instuctions: This is a 3 hour examination. In the problems below, all rings are commutative with identity. This is a closed book exam, also no notes, searching the web, or otherwise consulting external sources. Good luck!

1. Let G be a group of order 108. Show that G has a normal subgroup of order 9 or 27.

2. Let R be a ring, and let D be the set of all $x \in R$ such that x is a zero divisor or $x = 0$. Show that D is a union of prime ideals. (Hint: consider the set of all ideals contained in D. Show that contains maximal elements and every maximal element of is prime.)