Algebra Qualifying Examination, Fall 2019

Instuctions: This is a 3 hour examination. In the problems below, all rings are commutative with identity unless specified otherwise. This is a closed book exam, also no notes, searching the web, or otherwise consulting external sources. Good luck!

1. Let $P$ be a finite $p$-group. Show that $P$ is not cyclic if and only if $P$ has a quotient isomorphic to $\mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z}$.

2. Let $R$ be a commutative ring with unity.

   (a) Let $S$ be a non-empty saturated multiplicative set in $R$, i.e. if $a, b \in R$, then $ab \in S$ if and only if $a, b \in S$. Show that $R - S$ (the complement of $S$ in $R$) is a union of prime ideals.

   (b) Suppose that $R$ is a domain such that every nonzero prime [illegible] Suppose $A$ in the group $GL_N(\mathbb{C})$ [illegible] of finite order [illegible] is conjugate to a

   diagonal matrix.

   (b) If $F$ is an algebraically closed field and $A \in GL_N(F)$ is of finite order, is